SOLAR CONVECTION AS THE PRODUCER OF MAGNETIC BIPOLES

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ABSTRACT

A mechanism for the *in situ* formation of bipolar magnetic configurations, alternative to the well-known rising-tube mechanism, is considered. Cellular magnetoconvection is simulated numerically. The initial magnetic field is assumed to be fairly weak and directed horizontally. A pattern of small-amplitude hexagonal cells is specified initially. A hexagonal convection cell can substantially amplify the magnetic field and impart bipolar configurations to it. Depending on the parameters, the amplified field may form either a simple pair of magnetic islands, opposite in polarity, or a more complex superposition of bipoles. In particular, very compact magnetic elements of strong field can form. The resulting pattern of magnetic-field evolution is in better agreement with observations than the rising-tube model.

Key words: magnetoconvection; magnetic bipoles; sunspot formation.

INTRODUCTION

In the mid-1960s, Tverskoy (1966) considered a simple kinematic model of a convection cell in a perfectly conducting fluid and found that magnetoconvection can produce bipolar configurations of the amplified magnetic field, like those typical of sunspot groups. The original presence of a flux tube of strong magnetic field is not needed for this process, and the topology of convection cells is, on its own, responsible for the basic property of the considered mechanism. As noted by Getling (2001) and will be demonstrated below, the pattern of the magnetic-field evolution controlled by this mechanism much better agrees with observations (Bumba, 1967) than the rising-tube scenario does.

A comprehensive study of the convective mechanism and a quantitative verification of its efficiency under solar conditions require numerical simulations of the evolution of three-dimensional flows and magnetic fields, based on the full system of MHD equations. Some results of such simulations were reported by Getling (2001). We present here some further developments of this study. At the current stage of investigation, we restrict ourselves to the Boussinesq approximation—see, e.g., Getling (1998). In other words, we assume density variations to be negligibly small in all terms in the equations except for the term proportional to the gravitational acceleration.

THE PROBLEM

We use a standard system of MHD equations in the Boussinesq approximation to simulate magnetoconvection in a plane horizontal layer $0 < z < d$ of a fluid with a finite electrical conductivity, heated from below. The lower and the upper boundary of the layer are assumed to be free-slip, impermeable, of perfect electrical conductivity, and kept at constant temperatures T_1 and T_2, respectively, so that $T_1 - T_2 = \Delta T$. We represent any variable f in the problem as the sum of its *unperturbed* (static) value f_0 and a flow-produced *perturbation*, which, in general, may even substantially exceed the unperturbed value. The unperturbed (initial) magnetic field H_0 is uniform and directed horizontally, along the x axis. The magnetic-field perturbation h is measured in units of H_0, and the temperature perturbation θ is measured in units of ΔT. We choose d as the unit length and the characteristic time $t_\nu = d^2/\nu$ of viscous dissipation on the scale d as the unit time (ν is the kinematic viscosity).

The nondimensional parameters

$$R = \frac{\alpha g\, \Delta T\, d^3}{\nu\chi}, \quad Q = \frac{H_0^2 d^2}{4\pi\rho_0\nu\nu_{\mathrm{m}}} = \frac{H_0^2 d^2 \sigma}{\rho_0 c^2 \nu},$$

$$P_1 = \frac{\nu}{\chi}, \quad P_2 = \frac{\nu}{\nu_{\mathrm{m}}} = \frac{4\pi\sigma\nu}{c^2}$$

(where α is the volumetric thermal-expansion coefficient of the fluid, χ its thermal diffusivity, σ its electrical conductivity, and ν_{m} its magnetic viscosity) are the Rayleigh number, Chandrasekhar number, normal Prandtl number, and magnetic Prandtl

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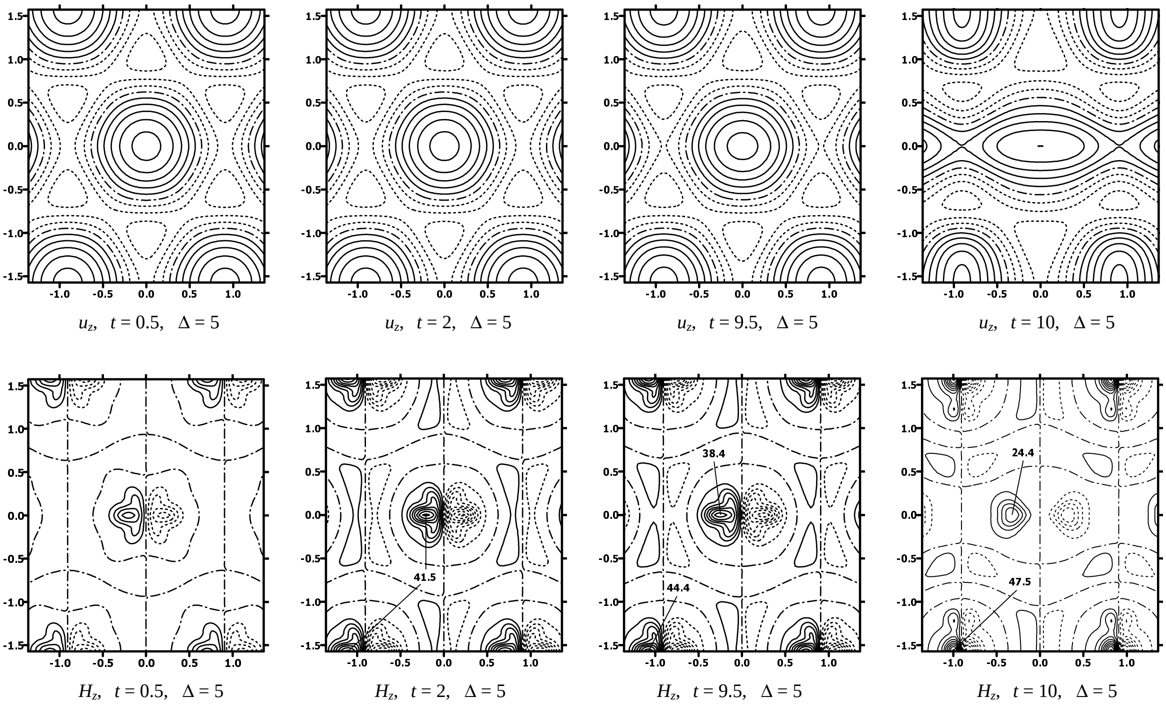

Figure 1. Evolution of the flow (top) and magnetic field (bottom) for $R = 3000 = 4.56R_\mathrm{c}$, $P_1 = 1$, $P_2 = 10$, $Q = 0.01$, $k_0 = 4 = 1.8k_\mathrm{c}$ ($R_\mathrm{c} = 657.5$ is the critical Rayleigh number at which convection sets in, and k_c is the critical wavenumber). Contours of the vertical components of the velocity, u_z, and magnetic filed, H_z, in the midplane $z = 1/2$ of the layer are shown with contour increment Δ. Solid lines: positive values; dash–dotted lines: zero values; dotted lines: negative values

number, respectively. They are the basic parameters of the problem. In addition, given the shape and orientation of the originally present convection cells, the magnetic-field evolution depends on their size (i.e., on the initially specified flow wavenumber).

We solve the problem numerically, using a Galerkin (spectral) technique and a fast Fourier transform procedure. The formulation of the problem and the techniques used in solving it are described in greater detail by Getling (2001).

In each run, a small-amplitude initial flow was imposed in the form of an x- and y-periodic pattern of Bénard-type hexagonal cells. Thus, the fundamental wavenumber of the pattern k_0 was specified.

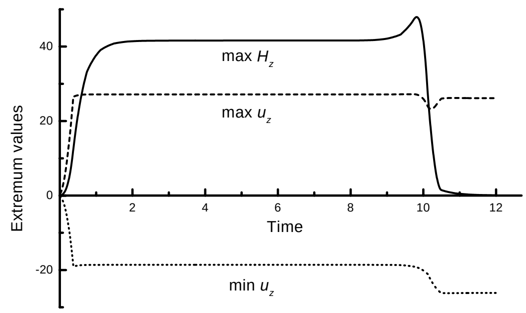

Figure 2. Time variation of extremum values of the vertical components of the magnetic field and velocity for $R = 3000 = 4.56R_\mathrm{c}$, $P_1 = 1$, $P_2 = 10$, $Q = 0.01$, $k_0 = 4 = 1.8k_\mathrm{c}$.

RESULTS

In all scenarios computed, the three-dimensional cellular flow ultimately underwent a transition to a two-dimensional roll flow, most typical of convection in uniform layers. After such a transition, the magnetic field decays. We shall not discuss here the relevance of this effect to solar magnetoconvection and will only be interested in that stage of the process when the flow is three-dimensional. Here, we describe three scenarios of flow and magnetic-field evolution.

The first one corresponds to moderate values of R and P_2 and a fairly small value of Q (Figs. 1, 2). It can be called the *basis* scenario, since its main features are typical of regimes with a "quiet" development of the process and a relatively simple magnetic-field structure. By the time $t \approx 0.5$, the cellular flow reaches a steady state, which persists until $t \approx 9$. During the interval $0 < t < 2$, the magnetic field is amplified (the *growth stage*), and bipolar configurations—pairs of compact magnetic islands—develop in the central parts of the convection cells. At $t \approx 2$, a *steady-state stage* ensues, with a magnetic-field strength within the islands of about 41.5 (in units of H_0). Between $t = 8$ and 9, a transition to a two-dimensional roll flow begins. In the case considered, at a fundamental wavenumber of $k_0 = 4$, the upwellings located at

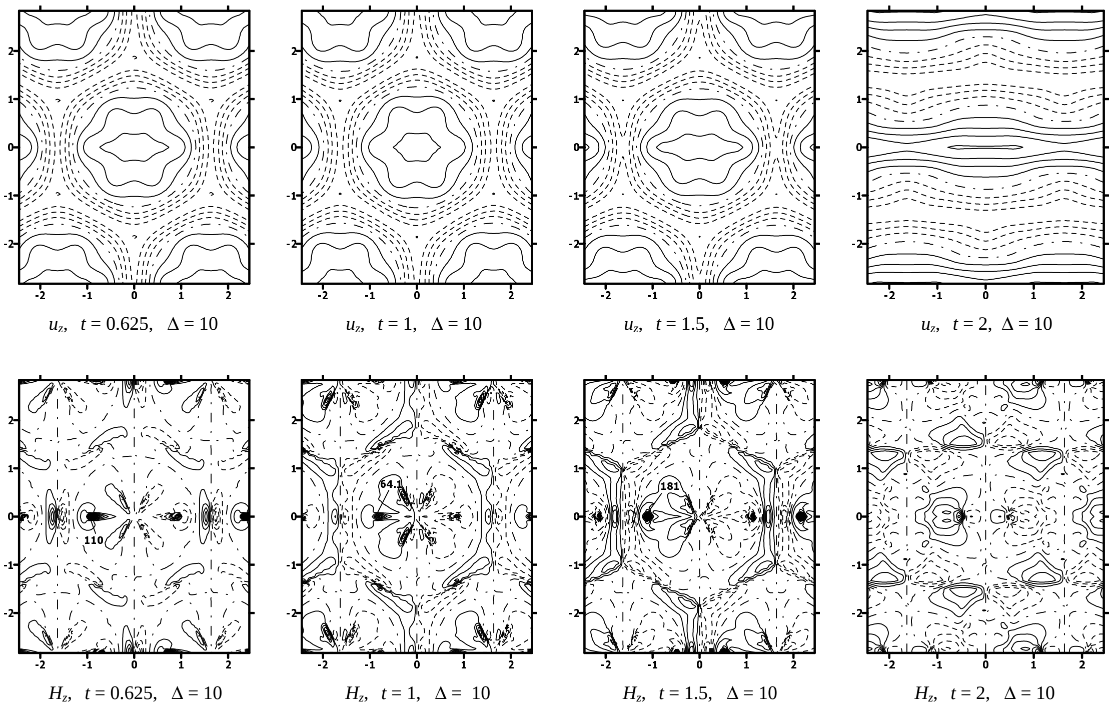

Figure 3. Same as in Fig. 1 but for $R = 5000 = 7.6R_{\rm c}$, $P_1 = 1$, $P_2 = 30$, $Q = 1$, $k_0 = 2.22 = k_{\rm c}$.

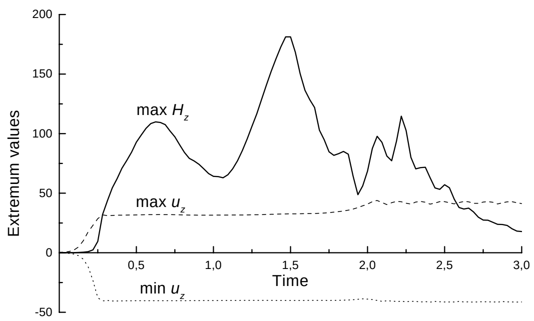

Figure 4. Same as in Fig. 2 but for $R = 5000 = 7.6R_{\rm c}$, $P_1 = 1$, $P_2 = 30$, $Q = 1$ $k_0 = 2.22 = k_{\rm c}$.

$y = \pm\pi/2, \pm 3\pi/2, \ldots$ are compressed by the interspacing downwellings, and the magnetic field in the upwelling regions is additionally amplified (the *compression stage*). At the same time, the upwellings located at $x = 0, \pm\pi, 2 \pm \pi, \ldots$ merge, and the magnetic field weakens there. The effect of magnetic-field compression is reflected by the peak in the curve of $\max H_z$ in Fig. 2 near $t = 9.9$, whose height is 49.1. Ultimately, the magnetic-field *decay stage* results in the recovery of the initial field, with a simultaneous formation of a roll flow.

The second scenario (Figs. 3, 4) is characterized by larger R, P_2, and Q; in addition, the fundamental wavenumber k_0 is equal to the critical one, $k_{\rm c}$. The evolution is more complex in this case, without a steady-state stage in the variation of the magnetic field. At $k_0 = k_{\rm c}$, the transition to rolls does not pass a compression stage. During the decay stage, the amplified magnetic field is fragmented, and its remnants survive relatively long. In each convection cell, a pair of very compact magnetic elements develops, where the magnetic flux is highly concentrated. These elements grow rapidly and reach a wide spatial separation very early. A significant magnetic-flux accumulation at the peripheries of the cells can be noted.

If we reduce the hydrodynamic Prandtl number, the magnetic elements become even smaller, with even higher flux concentration (the third scenario, computed tentatively, with a possibly incomplete spectrum; Figs. 5, 6). The time during which magnetic remnants persist is much longer than the width of the main $\max H_z$ peak in Fig. 6. In this case, as in the second scenario, multipolar configurations can also be formed at certain stages of the process.

In any particular case, the efficiency of the convective mechanism can be measured by the maximum achieved nondimensional H_z and by the ratio γ of the maximum values of the magnetic- and kinetic-energy densities $E_{\rm m}$ and $E_{\rm k}$:

$$E_{\rm m} = \frac{H_0^2\,(\max H_z)^2}{8\pi}, \quad E_{\rm k} = \frac{\rho_0\,(\max|u_z|)^2}{2}\frac{\nu^2}{d^2},$$

$$\gamma = \frac{E_{\rm m}}{E_{\rm k}} = \frac{Q\,(\max H_z)^2}{P_2\,(\max|u_z|)^2}.$$

In the first scenario, H_z reaches 49.1 near $t = 9.9$. At this time, $\gamma = 0.003$. The second scenario yields main-peak values of $\max H_z = 181.3$ and $\gamma = 0.68$ at $t = 1.47$. At time $t = 0.625$, when the first peak of $\max H_z$ is reached, these quantities are 109.9 and 0.25, respectively. Finally, the third scenario is characterized by $\max H_z = 688.1$ and $\gamma = 0.88$ at the main peak ($t = 0.266$).

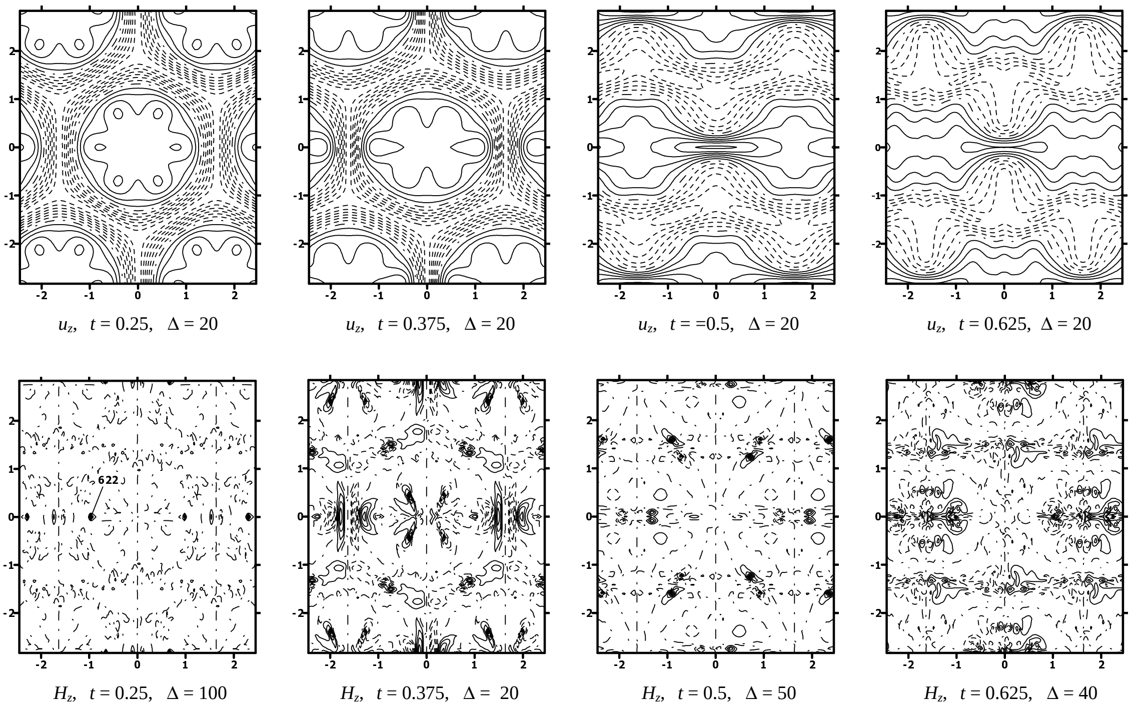

Figure 5. Same as in Fig. 1 but for $R = 5000 = 7.6R_{\rm c}$, $P_1 = 0.3$, $P_2 = 30$, $Q = 1$, $k_0 = 2.22 = k_{\rm c}$.

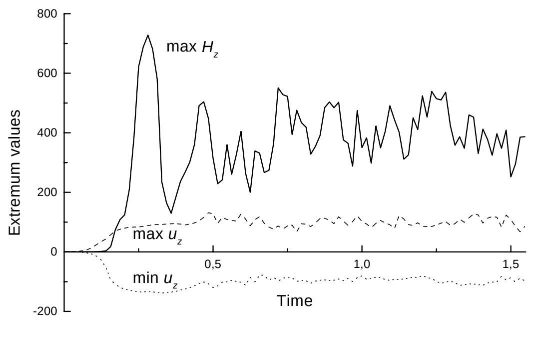

Figure 6. Same as in Fig. 2 but for $R = 5000 = 7.6R_{\rm c}$, $P_1 = 0.3$, $P_2 = 30$, $Q = 1$, $k_0 = 2.22 = k_{\rm c}$.

DISCUSSION

Thus, our computations confirm the qualitative predictions based on the model of Tverskoy (1966). The above-described convective mechanism of magnetic-field amplification and structuring is very efficient and does not require strong initial magnetic fields. Since its main properties are determined by the flow-cell topology, it can operate on various spatial scales. While regular supergranular cells may form small magnetic elements, an especially large and intense cell, which encompasses layers deeper than usual and is threaded with a weak seed magnetic field, could produce the magnetic fields of sunspot groups. Diverse initial conditions can ensure the generation of diverse configurations of the amplified magnetic field.

Our notion of the magnetic-field amplification process is free of the contradictions with observations that are inherent in the rising-tube model:

(1) If a tube of a strong magnetic field emerges, it will completely break down the pre-existing convective velocity field. In contrast, the actually observed flow pattern normally remains virtually intact in the process of local magnetic-field growth; the magnetic field only gradually *seeps* through the photosphere, retaining its conformity with the velocity field (Bumba, 1967). It is such a situation that should take place if the magnetic field is produced by the convective mechanism.

(2) Some dramatic effects of the tube emergence, such as plasma streams spreading from the site above the rising tube, should be especially impressive but have never been observed. They are not implied by the convective mechanism.

(3) The emerging magnetic field itself, strong and mainly horizontal, would be directly observed in the photosphere as a prominent feature of the process, but nothing of the sort actually takes place. If the convective mechanism operates, similar effects should not be expected.

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